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THE MAIN RING AND THE ENERGY DOUBLER AS SOURCES OF SYNCHROTRON RADIATION

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Comment

I wrote the following note in March 1972. Since then I let it rest in my files. I decided to resume it now and to publish it as an internal note after I have noticed recently some interest on this subject, especially during the POPAE and the colliding beam studies. The reader should pay attention to the fact that the wording refers to a situation which is obviously out of date.

One obvious application of the synchrotron radiation is the measurement of the beam size in the colliding beam situation.

THE SYNCHROTRON RADIATION

A. G. Ruggiero

INTRODUCTION

This is a proposal of a series of experiments to be performed with the proton beam accelerated in the main ring, (a) in the near future, at the energy of 200 GeV, (b) sometime later, at 500 GeV when this energy can be reached, and, (c) at 1000 GeV with the energy doubler.

Even at the low energy of 200 GeV the protons radiate such large amounts of power that we should be able to detect and investigate. The power is radiated in the form of electromagnetic waves and is generally known as "synchrotron radiation." The spectrum of this radiation depends on the energy of the protons. At 200 and 500 GeV the radiation is respectively in the Hertz short wavelength range (~0.1 mm) and in the infrared (~0.01 mm). At 1000 GeV a large fraction of the power is radiated in the "visible" and is so intense that a filtering system might be required.

The theoretical study of the "synchrotron radiation" has been developed by several authors (1-5) who provided, on the basis of the clssical electrodynamics, the formulae for the calculation of the frequency distribution, spatial distribution and polarization. Experimental verifications of these formulae have been tempted by several physicists (6-11) but only for the case where the sources of the radiation were <u>electrons.</u>

We suggest, here: (1) to observe for the "synchrotron radiation" from ultrarelativistic <u>protons</u>, and (2) to investigate the nature of the radiation (spectrum, spatial distribution and polarization) to check that the two kinds of radiation, from electrons and protons, are identical.

THE SPECTRUM OF RADIATION

1. Let us consider the case of a proton traversing a bending magnet. We suppose the magnetic field $\vec{B}$ is uniform around the trajectory of the particle and perpendicular to the motion.

Hence, the trajectory is an arc of circle of radius R given by

$$R = \frac{cp}{eB}$$

where

e = the proton charge = 4.8×10^{-10} c.g.s.u.

c = the velocity of light = 3×10^{10} cm s^{-1}

p = the momentum of the particle.

Denoting the velocity of the proton by v and $\beta = v/c$, the proton traverses the bending magnet with angular velocity

$$\omega_o = \beta c/R.$$

The radius R is independent of p. The angular velocity ω_o, also, is independent of p for ultrarelativistic protons ($\beta \sim 1$).

In our case it is

$$R = 748 \text{ m} \qquad \text{and} \qquad \omega_o = 4.01 \times 10^5 \text{ s}^{-1}.$$

2. The instantaneous power radiated by the proton while traversing the bending magnet is

$$P = \frac{2}{3}\,\omega_o\,\frac{e^2}{R}\,\gamma^4$$

where γ is the ratio of the proton total energy to the proton rest energy.

The power radiated in a unit wavelength range about λ is

$$P(\lambda) = \frac{3^{5/2}}{16\pi^2} \frac{\text{[illegible]}}{R^3} \gamma^7 G(y)$$

$$y = \frac{\lambda_c}{\lambda}$$

$$\lambda_c = \frac{4\pi R}{3\gamma^3}$$

$$G(y) = y^3 \int_y^\infty K_{5/3}(\eta)\,d\eta$$

where $K_{5/3}$ is the modified Bessel function of fractional order (5/3). The function $G(y)$ is plotted in Fig. 1.

It can be verified that

$$\int P(\lambda)\,d\lambda = P.$$

All these formulae apply (a) for ultrarelativistic protons with $\gamma >> 1$, (b) for wavelength λ very much smaller than the magnet length, and (c) far away from the edges of the magnet.

For our practical purposes it is

$$P(\lambda) = 1.63 \times 10^{-24} \gamma^7 G\left(\frac{\lambda_c}{\lambda}\right) \quad \text{erg}\cdot\text{s}^{-1}\cdot\text{cm}^{-1}$$

$$P = 8.22 \times 10^{-19} \gamma^4 \ \text{erg}\cdot\text{s}^{-1}$$

$$\lambda_c = 3.13 \times 10^{13} \gamma^{-3} \ \text{A}^\text{o}$$

Some values are reported in Table I. λ_{max} is the wavelength at which $P(\lambda)$ has a maximum, and $P_{max} = P(\lambda_{max})$. It is

$$\lambda_{max} \simeq 0.42\ \lambda_c.$$

Table I

γ	200	500	1000
$P(erg \cdot s^{-1})$	1.32×10^{-9}	5.14×10^{-8}	8.22×10^{-7}
$\lambda_c (A^o)^*$	3.91×10^{6}	2.50×10^{5}	3.13×10^{4}
$\lambda_{max} (A^o)^*$	1.64×10^{6}	1.05×10^{5}	1.31×10^{4}
$P_{max} (erg \cdot s^{-1} \cdot cm^{-1})$	2.65×10^{-8}	1.62×10^{-5}	2.07×10^{-3}

$^* 1A^o = 10^{-10} m.$

3. Let us consider the power radiated by a bunch of 10^{10} protons traversing a bending magnet.

a. At $\gamma = 200$, the radiation is in the upper limit of the range of the Hertz waves, with a wavelength of the order of a few tenths of a millimeter. The power is of the order of few μW. Part of the spectrum (with a wavelength of about one or a few millimeters), probably can be observed with a microwave technique device (cavities, wave guides, ...). The main part, at shorter wavelength, might be observed with a thermocouple device.

b. At $\gamma = 500$, the radiation is all in the far infrared zone. The power is of the order of 50 μW. Also, this radiation might be observed with a thermocouple device.

c. At $\gamma = 1000$, somewhat more than 50% of the spectrum is in the "very near" infrared zone with a power of about 0.5 mW. But a large fraction of the spectrum is in the "visible" and the power radiated in this zone is about 0.3 mW. As we shall see later, the "visible" radiation is almost entirely radiated in an angle of ~2 mrad. Thus, looking tangentially to the proton trajectory, the radiation observed by naked eye should look as coming from a "red spot" with an intensity equivalent to that radiated at the same time by several tens of lamps of 40 W each.

ANGULAR DISTRIBUTION OF RADIATION

The radiation from a single particle is extremely coherent. Nevertheless, in practice, the coherence of the radiation from a bunch of particles can be observed only in the vertical plane. The finite longitudinal dimension of the bunch and the motion on the radial plane mask completely the coherence of the radiation on the horizontal plane.

If we denote by ψ the angle between the direction of emission and the orbital plane, the power radiated by a single particle at the frequency with wavelength λ and around the angle ψ is

$$P(\psi,\lambda) = \frac{3}{2\pi}\frac{e^2}{R}\left(\frac{\lambda_c}{\lambda}\right)^2 \frac{c}{\lambda^2}\gamma^2 \left(1+\gamma^2\psi^2\right)^2 \times$$

$$\times \left[K^2_{2/3}(\xi) + \frac{\psi^2\gamma^2}{1+\psi^2\gamma^2}K^2_{1/3}(\xi)\right] = \frac{27}{32\pi^3}\frac{e^2 c}{R^3}\gamma^8 \, F\left(\gamma\psi, \frac{\lambda_c}{\lambda}\right)$$

where

$$\xi = \frac{\lambda_c}{2\lambda}\left(1 + \psi^2\gamma^2\right)^{3/2}$$

and $K_{1/3}$, $K_{2/3}$ are the modified Bessel function, respectively, of order 1/3 and 2/3.

The total angular distribution is obtained by integrating $P(\psi,\lambda)$ over λ:

$$P(\psi) = \int P(\psi,\lambda)\, d\lambda$$

$$= \omega_o \frac{e^2}{R}\gamma^5\left(1+\gamma^2\psi^2\right)^{-5/2}\left(\frac{7}{16} + \frac{5}{16}\frac{\gamma^2\psi^2}{1+\gamma^2\psi^2}\right)$$

$$= \omega_o \frac{e^2}{R}\gamma^5 \, F(\gamma\psi).$$

The function $F(\gamma\psi)$ is plotted in Fig. 2. The radiation is almost all emitted in $\sim\pm\gamma^{-1}$ radians angle.

The function $F\left(\gamma\psi, \frac{\lambda_c}{x}\right)$ is plotted in Fig. 3 for $\lambda_c/\lambda = 5$, which for $\gamma = 10^3$ corresponds to a radiation in the red.

Some data are reported in Table II.

Table II

γ	200	500	1000
ψ_1 (mrad)	4.05	1.62	0.81
ψ_2 (mrad)	2.00	0.80	0.40
P_1 (erg s^{-1} $mrad^{-1}$)	1.73×10^{-10}	1.69×10^{-8}	5.39×10^{-7}
P_2 (erg s^{-1} cm^{-1} $mrad^{-1}$)	3.05×10^{-9}	4.65×10^{-6}	1.19×10^{-3}

ψ_1 is the angle at which the angular distribution $P(\psi)$ decays at the value 1/e times the maximum. ψ_2 is the same angle but refers to the differential distribution $P(\psi,\lambda)$ for $\lambda_c/\lambda = 5$.

P_1 is the total power radiated in a unit angle at $\psi = 0$. P_2 is the power radiated with $\lambda = \lambda_c/5$ in a unit angle at $\psi = 0$.

Experimental observation of the spatial distribution of the radiation is easy at the higher energy of 10^3 GeV, since "optical" techniques can be used. But we do not know, at the present moment, if there is any technique for the observation of the angular distribution at lower energy. Indeed, if the power radiated is enough to be detected with an integrated measurement, it might be not enough for more detailed observations, also, because of the larger wavelengths involved.

POLARIZATION

The frequency and angular distributions have been fully investigated by J. Schwinger in a now classic paper (3). The method this author used was not suitable to derive the polarization status of the "synchrotron radiation." For this purpose, it is necessary to inspect the distribution of the electric vector $\vec{E}$ instead of its intensity. This was done by K. C. Westfold in a more recent paper (5). Westfold proved the radiation is elliptically polarized with the axis of polarization one parallel and the other perpendicular to the orbit plane. In the following E_{11} denotes the component of $\vec{E}$ parallel to the orbit plane and E_1 the component perpendicular to the same plane. We have

$$E_{11} = \frac{2e}{R\gamma^2\pi\sqrt{3}} (1+\gamma^2\psi^2) \; K_{2/3}(\xi)$$

$$E_1 = \frac{2e}{R\gamma^2\pi\sqrt{3}} \gamma\psi(1+\gamma^2\psi^2)^{1/2} K_{1/3}(\xi).$$

At $\psi = 0$ the polarization is linear, the perpendicular component being zero. As ψ moves to $\pi/2$ the polarization becomes circular. This is better described by the ratio

$$r = \frac{E_{11}}{E_1} = \frac{(1+\gamma^2\psi^2)^{1/2}}{\gamma\psi} \frac{K_{2/3}(\xi)}{K_{1/3}(\xi)} .$$

One has to observe what we can measure is not the amplitude of the electric field but its intensity, namely E_{11}^2 and E_1^2, or r^2. It is easily verified that $E_{11}^2 + E_1^2$ is just the angular distribution $P(\lambda,\psi)$ described in the previous section.

The following two functions

$$F_{11} = (1+\gamma^2\psi^2)^2 \; K_{2/3}{}^2(\xi)$$

$$F_1 = \gamma^2\psi^2(1+\gamma^2\psi^2)K_{1/3}^2(\xi)$$

are plotted in Fig. 4 versus $\gamma\psi$ for $\lambda_c/\lambda = 5$.

EXPERIMENT SET-UP

The experiment for the observation of the "synchrotron radiation" might be performed in a way similar to that described in Ref. (11)

We need to draw a line tangent to the orbit in a bending magnet which is followed by a drift space of large length, as shown in Fig. 5. Across this line we shall place the radiation detector.

Among all the bending magnets in one sector of the main ring (or of the doubler), only three are before a long drift section. They are:

a. The third dipole in the "long straight section cell." It is followed by four quadrupoles for a total length of 8.8 m, and then by the long straight section of 50.8 m.

b. The last dipole of the "normal cell" before the "long straight section cell." It is followed by a length of 1.6 m with one quadrupole included, and by a drift space 9.3 m long.

c. The dipole preceding the "medium straight section." It is followed by a length of 2.4 m with one quadrupole, and by the "medium straight section" 14.859 m long.

In all these cases the observation line is as that outlined in Fig. 5. We have four sectors (denoted by Roman numbers in Fig. 5):

I. The bending sector, where the protons radiated energy. It extends over an angle ε from the point of irradiation 0 to the edge A of the dipole.

II. The quadrupole sector between A and B of length ℓ_o. This sector should be as short as possible to avoid large dispersion of the radiation.

III. The drift space between B and C of length ℓ. It is in this sector that the radiation crosses the pipe wall (of material transparent to the radiation) and hits the detector.

IV. All the space behind the point C cannot **be used.**

Our choice is magnet (c). Indeed, magnet (a) is followed by too many quadrupoles, and the drift space after magnet (b) is too short.

The exit point S and the position of the detector, as well as the amount of radiation detected, depends on the observation angle ε as explained below.

Let us introduce the following notation:

d = vacuum chamber (horizontal) size

$2a$ = detector "aperture" on the horizontal plane

b = distance of the detector center from the pipe wall

x_o = distance between T_1 and A

x_1 = distance between T_2 and B

x = distance of S from B'.

It is

$$x_o = \frac{1}{2} R\varepsilon^2 \qquad \text{and} \qquad x_1 = x_o + \ell_o\varepsilon$$

s and x_1 are plotted versus ε in Fig. 6.

For a good observation of the radiation the aperture a of the detector should be large and the detector itself as close as possible to the pipe wall. We take

$$2a = 1 \text{ cm}, \qquad b = 5 \text{ cm} \qquad \text{and} \qquad d = 10 \text{ cm}.$$

The length SD' and the sum s+SD' are also reported in Fig. 6. As we can see from this figure a good angle of observation is

$$\varepsilon = 7.75 \text{ mrad.}$$

The amount of radiation that goes through the detector depends on the aperture a. If the "window" of the detector is centered and perpendicular to the direction of angle ε, all the power radiated with angle around ε between ε_A and ε_B is collected, ε_A and ε_B being, respectively, the directions through the edges R_1 and R_2 of the "window." The two angles ε_A and ε_B are plotted versus ε in Fig. 6.

For $\varepsilon = 7.75$ mrad, it is $\varepsilon_A - \varepsilon_B = 0.45$ mrad. That is equivalent to say that we can observe the power radiated by all the protons distributed over an arc 0.34 m long. At the design intensity of 5×10^{13} particles, those protons are in average 2.7×10^9. As we have seen previously, there is a lot of radiation from this number of particles.

We conclude with the reminder that the "synchrotron radiation" observation might be useful for the purpose of monitoring with high precision the number of particles being accelerated.

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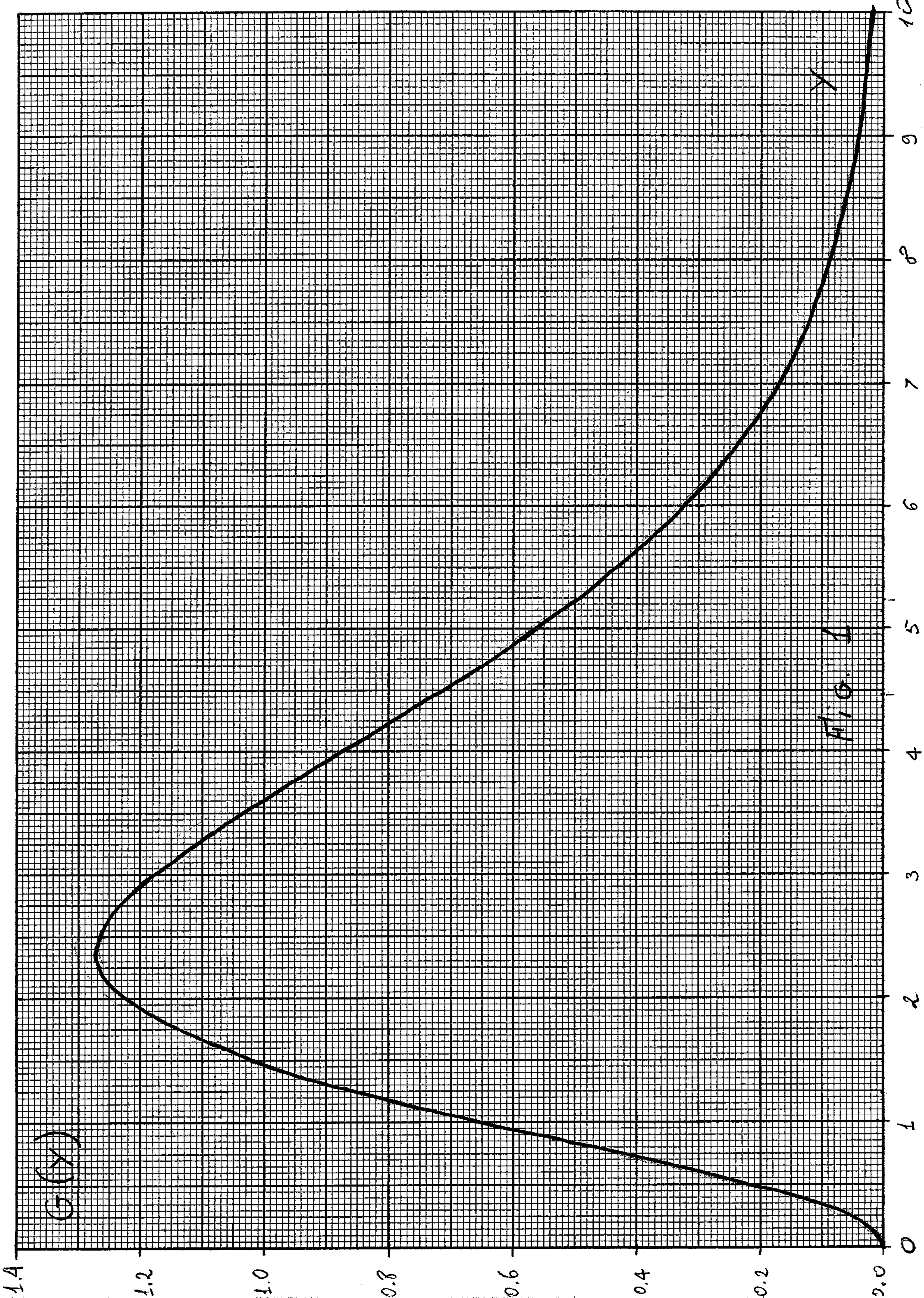

FIG. 1

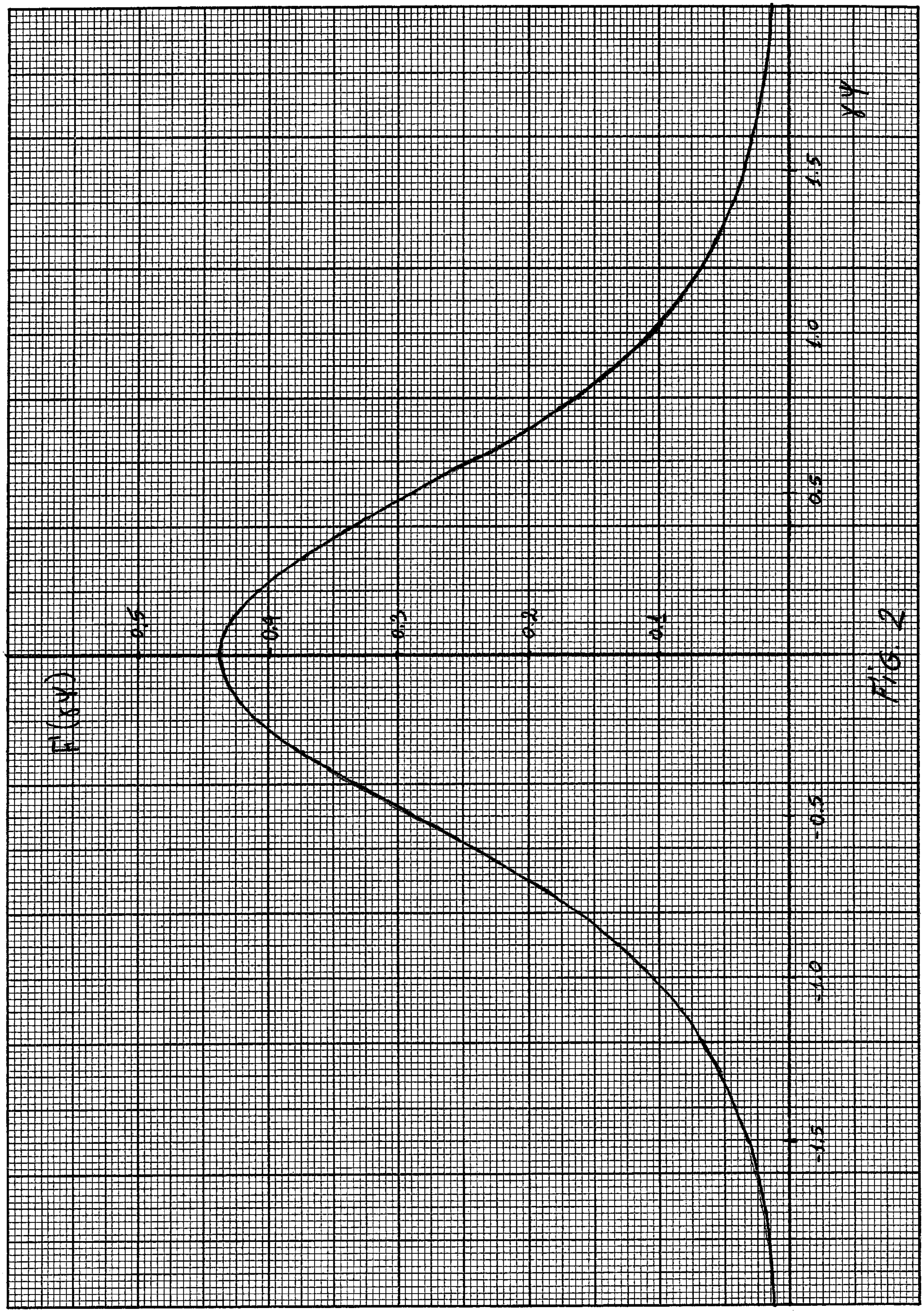

FIG. 2

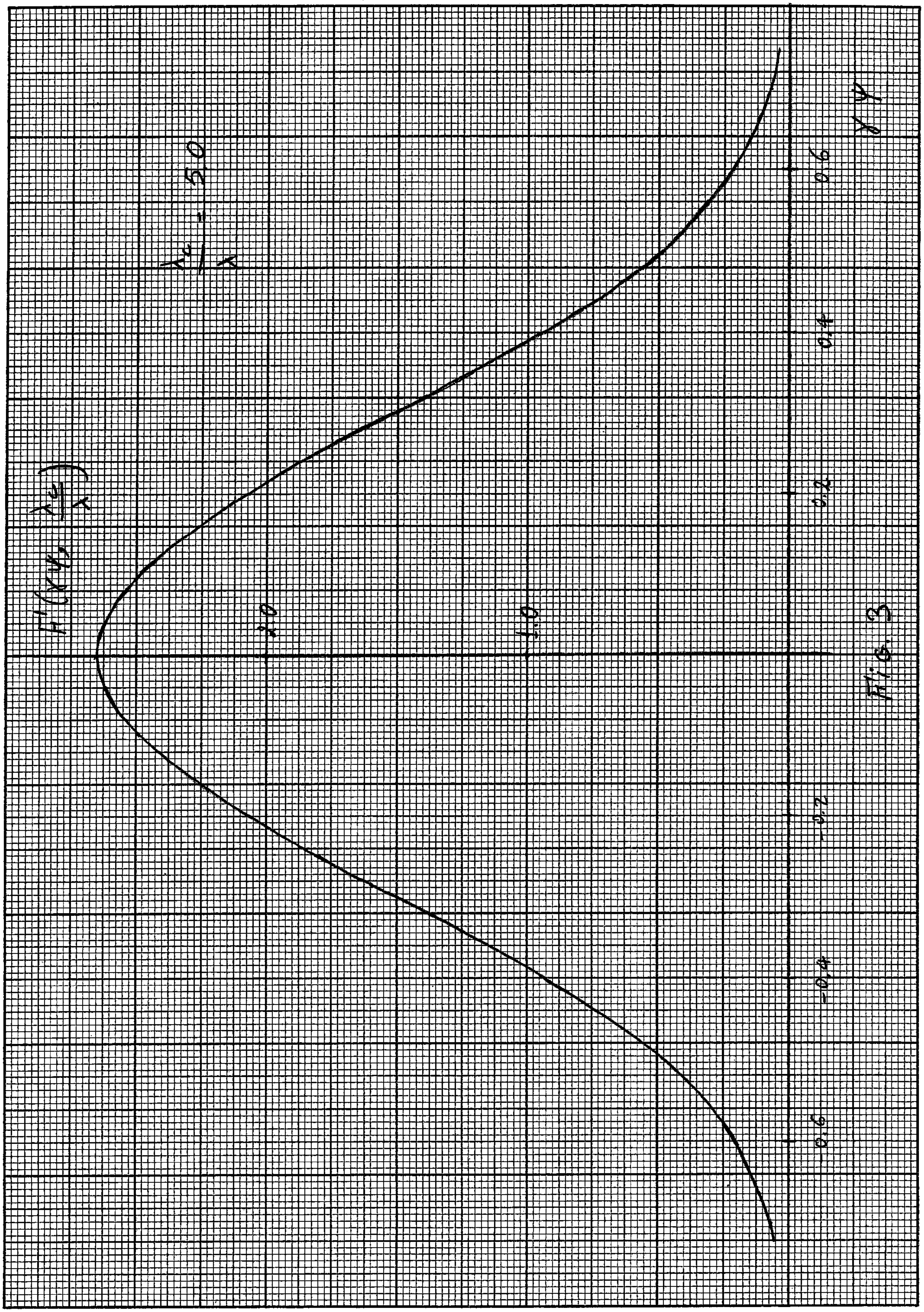
$F'(\gamma\psi, \frac{\lambda_c}{\lambda})$
$\frac{\lambda_c}{\lambda} = 5.0$
2.0
1.0
0.6
0.4
0.2
-0.2
-0.4
-0.6
$\gamma\psi$

FIG. 3

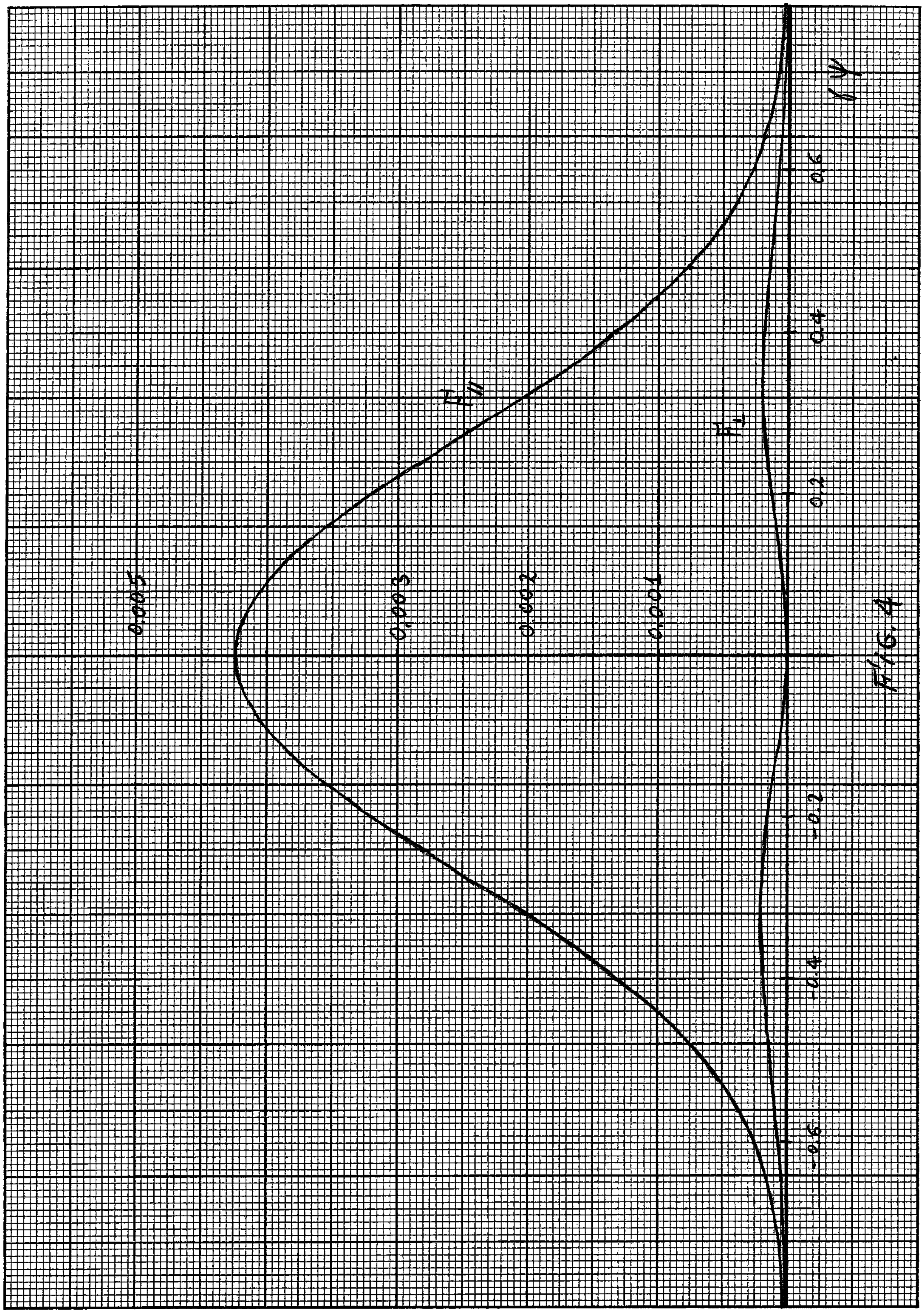

FIG. 4

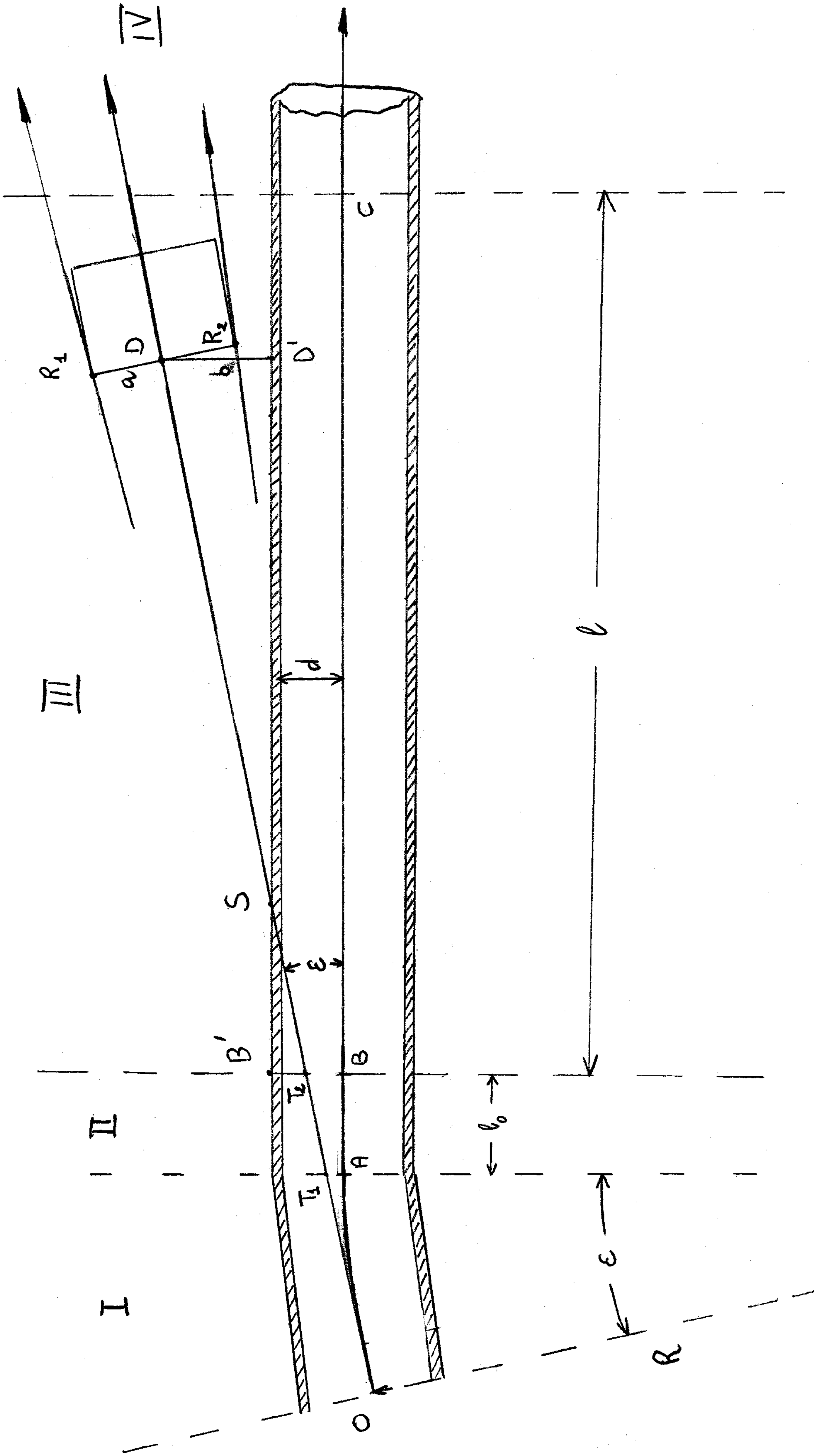

FIG. 5

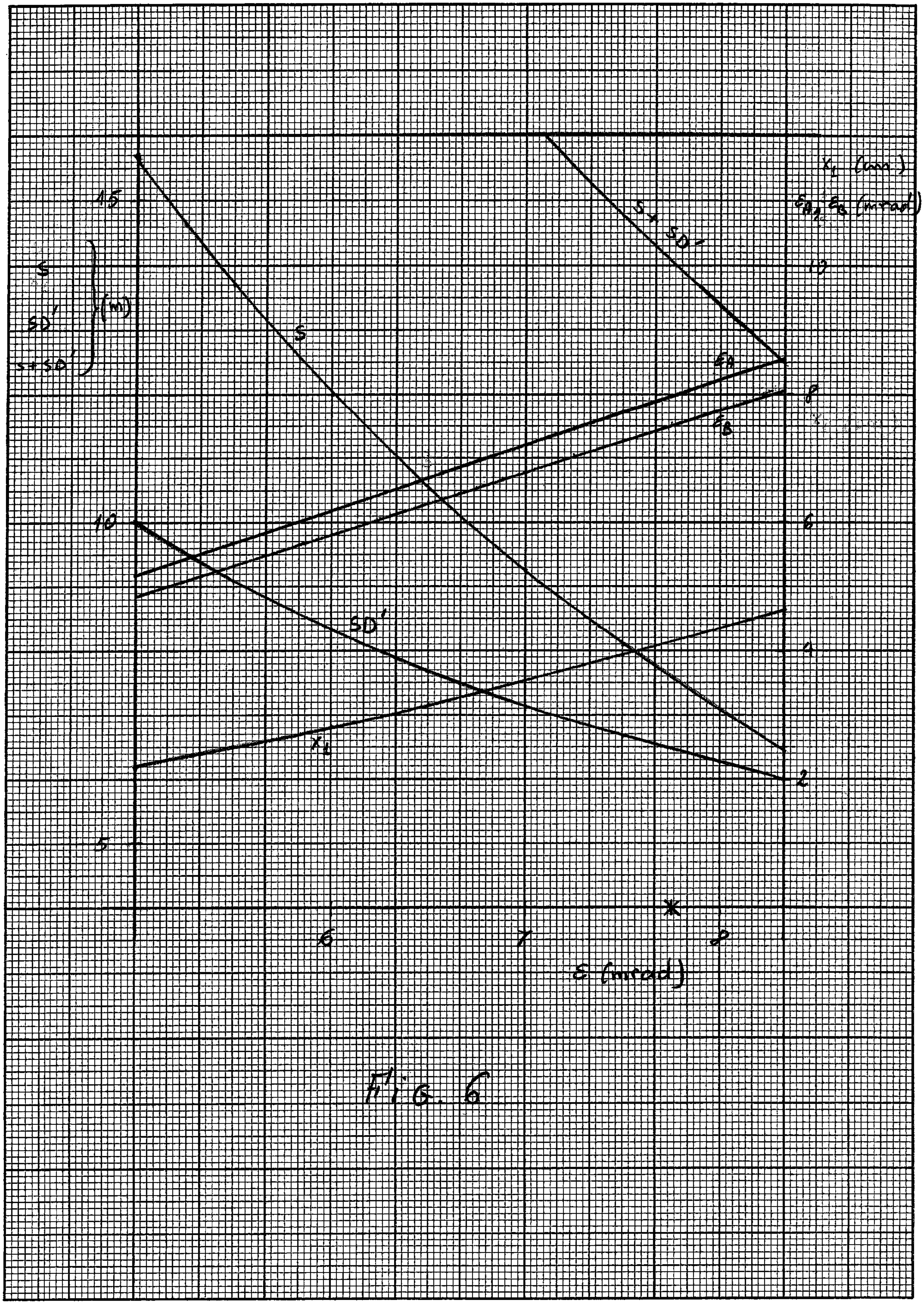
15
10
5
S
SD'
s+SD'
(m)
S
S+SD'
SD'
ε_A
ε_B
X_1
X_1 (mm)
ε_A, ε_B (mrad)
10
8
6
4
2
6
7
8
ε (mrad)

FIG. 6